

Ch. 3: Pair of Linear Equations in two variables

Linear Equations in Two Variables:

An equation of the form $ax + by + c = 0$, where a , b and c are real numbers and a and b are not both zero, is called a linear equation in two variables.

For example: $2x + 3y + 5 = 0$, $x - 5y - 8 = 0$ are the linear equations in two variables x and y .

Pair of linear equations in two variables:

The general form of linear equations in two variables x and y if

$$\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}$$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and

$$a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0.$$

A pair of values of variables x and y satisfying each one of the pair of equations is called a solution of the given pair of linear equations.

Solving simultaneous linear equations:

① Graphical method

(ii) Algebraic method.

(1) Graphical method :-

(i) Obtain the pair of linear equations.

$$a_1x + b_1y + c_1 = 0 \text{ --- (I)}$$

$$\text{and } a_2x + b_2y + c_2 = 0 \text{ --- (II)}$$

(ii) Find at least two solutions for each of the two equations by assuming value of one variable and then calculating the other variable.

(iii) Plot these points (solutions) of both equations in the same co-ordinate axes to get two straight-line, one for each equation.

Case I The two lines intersect at a point A then two equations have unique solution given by $x=a$, and $y=b$.

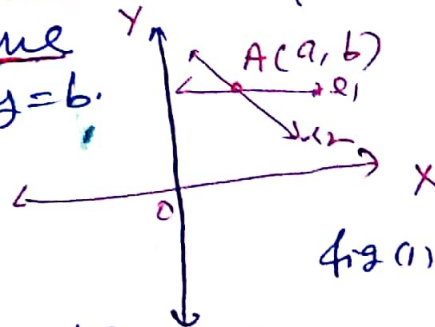


Fig (i)

Case II The two lines coincide each other Fig (ii). Then the two equations have infinitely many solutions. Equations are said to be consistent.

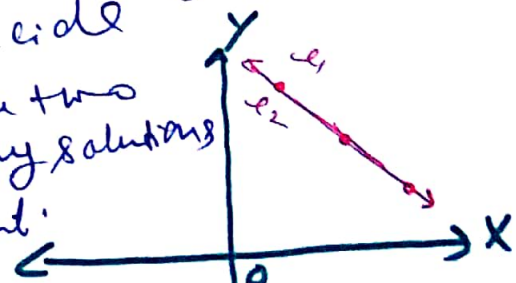
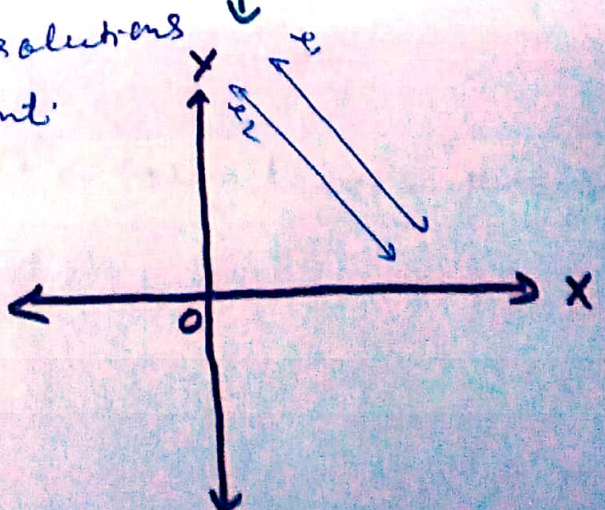


Fig (ii)

Case III The two lines are parallel to each other.

Then two equations have no solutions and are said to be inconsistent.



Ex 3.1 (MCERT)

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Let the age of Aftab be x years.
and the age of his daughter be y years.

7 years ago

$$\text{Age of Aftab} = (x-7) \text{ years}$$

$$\text{Age of his daughter} = (y-7)$$

A.T.Q.

$$(x-7) = 7(y-7)$$

$$x-7 = 7y-49$$

$$x-7y-7+49=0$$

$$x-7y+42=0 \quad \text{--- (I)}$$

After 3 years:

$$\text{Age of Aftab} = (x+3) \text{ years}$$

$$\text{Age of his daughter} = (y+3)$$

A.T.Q.

$$(x+3) = 3(y+3)$$

$$x+3 = 3y+9$$

$$x-3y+3-9=0$$

$$x-3y-6=0 \quad \text{--- (II)}$$

Graphical solution:

$$x-7y+42=0 \quad \text{--- (I)}$$

$$\Rightarrow x = 7y - 42$$

Put $y=6$

$$x = 7 \times 6 - 42$$

$$x = 42 - 42$$

$$\boxed{x=0}$$

Put $y=7$

$$x = 7 \times 7 - 42$$

$$x = 49 - 42 = 7$$

$$\boxed{x=7}$$

Put $y=8$

$$x = 7 \times 8 - 42$$

$$x = 56 - 42$$

$$\boxed{x=14}$$

x	0	7	14
y	6	7	8

For equation (11)

$$x - 3y - 6 = 0$$

$$x = 3y + 6$$

Let $y = 0$

$$x = 3 \times 0 + 6$$

$$x = 0 + 6$$

$$x = 6$$

Let $y = 1$

$$x = 3 \times 1 + 6$$

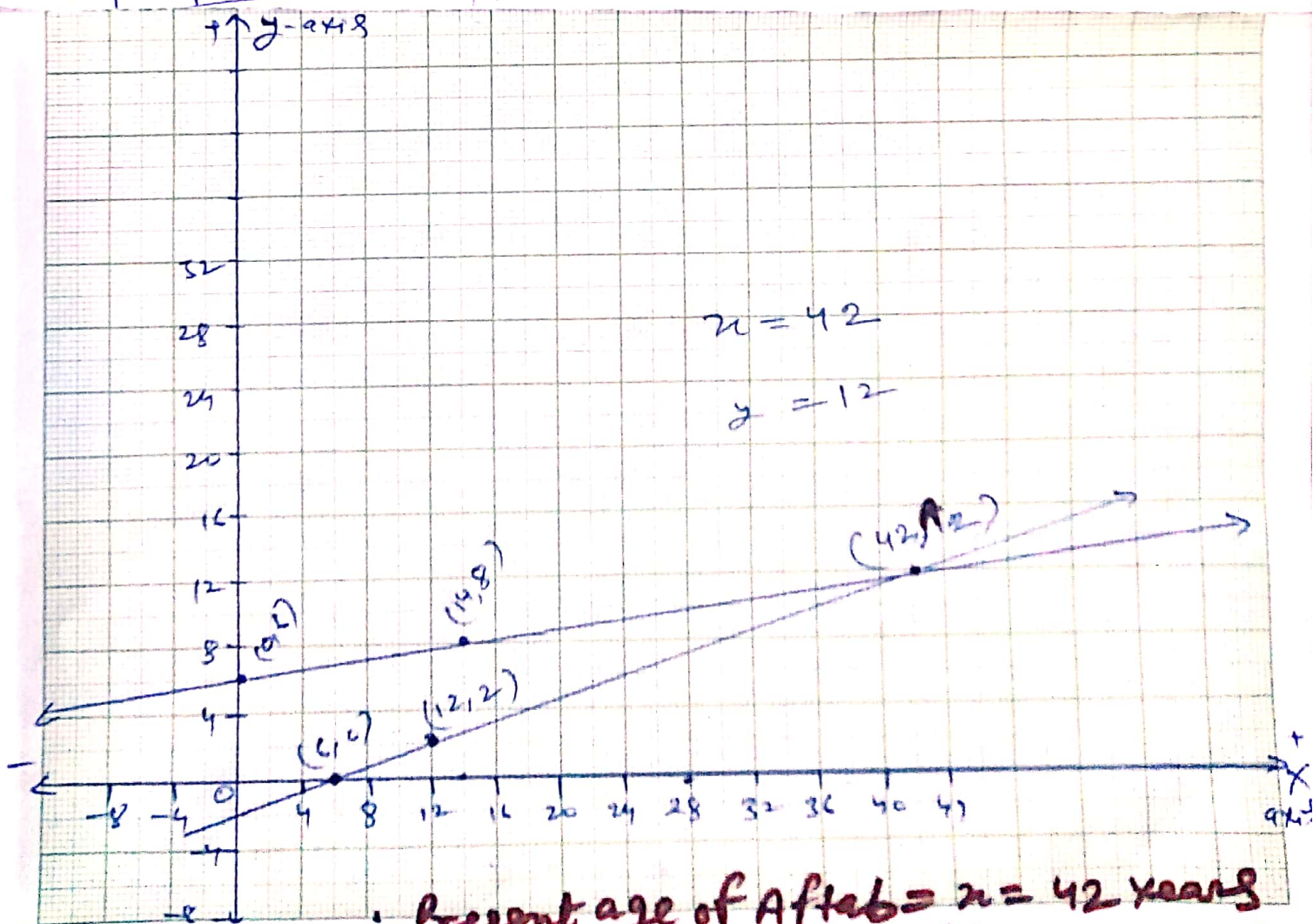
$$x = 9$$

Let $y = 2$

$$x = 3 \times 2 + 6$$

$$x = 12$$

x	6	9	12
y	0	1	2



\therefore Present age of Aftab = $x = 42$ years
" " his daughter = $y = 12$ years

Conditions for solvability of pair of linear equations:

(5)

A pair of linear equations

$$a_1x + b_1y + c_1 = 0 \quad \text{--- (i)}$$

$$a_2x + b_2y + c_2 = 0 \quad \text{--- (ii)}$$

(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (the lines intersecting each other and its solution is unique.)

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (the lines are coincident and equations has infinitely many solutions.)

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ The lines are parallel and system of linear equations has no solution.

Ex 3.2

Q.2

(i) $5x - 4y - 8 = 0 \quad \text{--- (1)}$

$7x + 6y - 9 = 0 \quad \text{--- (2)}$

Here

$$\frac{a_1}{a_2} = \frac{5}{7}, \quad \frac{b_1}{b_2} = \frac{-4}{6}, \quad \frac{c_1}{c_2} = \frac{-8}{-9}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

\therefore pair of linear equation intersect at a point and it has unique solution.

HOME WORK

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Ex 3.1 (NCERT)

Q.2 and 3.

Ex 3.2

Q.1,

Q.2 (ii) (iii)

Q.3 and Q.4

ASSIGNMENT

Q.1 Solve the following pair of equations graphically

(i) $x + 2 = 6$
 $x - 2 = 2$

(ii) $4x - y = 8$
 $3x + 2y = 12$

(iii) $x + y = 3$
 $2x + 5y + 12 = 0$

(iv) $2x + 3y = 8$
 $x + 4y = 9$

Q.2 For which value of k will the pair of equations $kx + 3y = k - 3$; $12x + ky = k$ have no solution?

Q.3 On comparing ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out following pair of linear equations intersect at a point, are parallel or coincident.

(i) $3x - 4y + 6 = 0$
 $x + 2y - 4 = 0$

(ii) $5x + 3y + 2 = 0$
 $15x + 9y + 6 = 0$